Functional renormalization group and ultracold fermions

Description of Bose-Einstein condensation without bosonic fields

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Frontiers of Hadron Pshysics: Brain circulation kickoff workshop

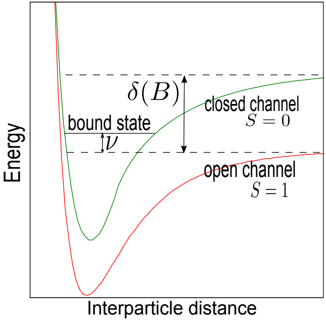
Collaborators: Tetsuo Hatsuda (RIKEN), Gergely Fejos (RIKEN)

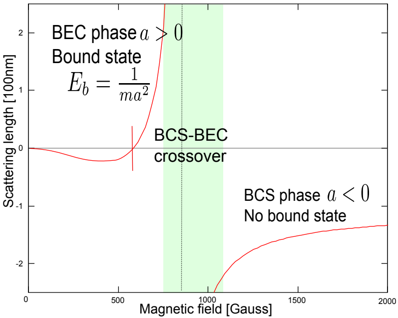
Cold atoms: BCS-BEC crossover

Ultracold atomic gases

What is ultracold atomic gases?

1. Ultracold: temperature $\sim$100nK
2. Dilute: number density $\sim 10^{11}$-$10^{14}\mathrm{cm}^{-3}$
3. Tunable interactions





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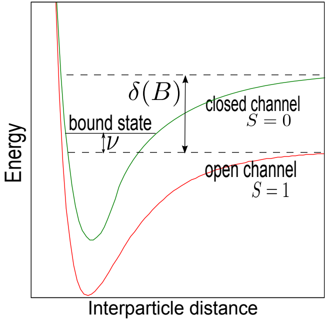
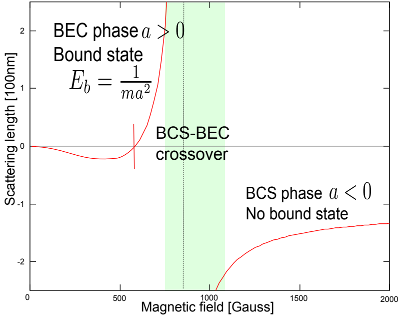
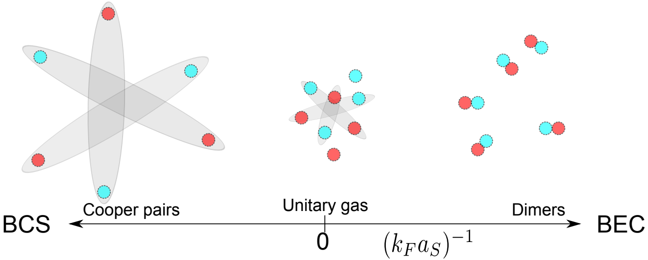




$\Rightarrow$ Strongly correlated systems can be established.

BCS-BEC crossover

System: two-component fermions with an attractive interaction

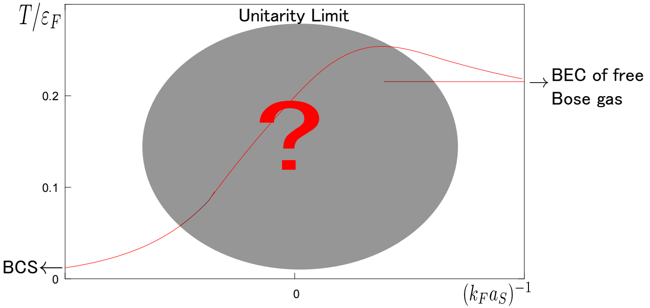


Important!

Two kinds of superfluid, BCS-like and BEC-like ones, are connected as the coupling a_S changes. (Eagles 69, Leggett 80)

Theoretical challenges in BCS-BEC crossover

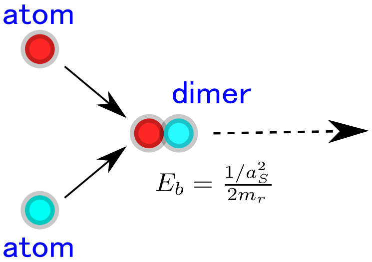
- Determination of T_c in the whole coupling region.
- Thermodynamic properties: especially at the unitarity limit $(k_F a_S)^{-1} = 0$.



(Nozieres and Schmitt−Rink 1985,...)

Formation of dimers

Positive scattering length regions ($a_S > 0$): bosonic bound states (dimers)



$$E_b = \frac{1/a_S^2}{2m_r}$$

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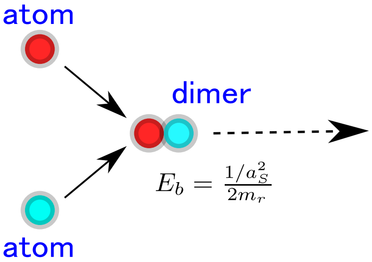


Purpose of this talk

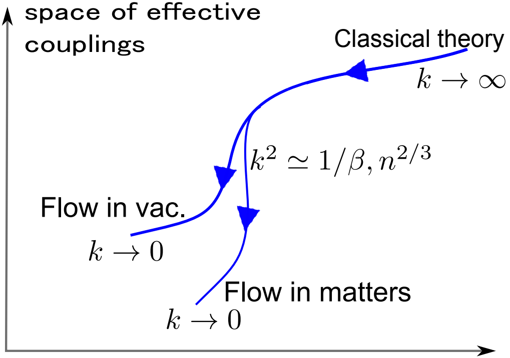
Describe BEC of dimers without auxiliary field methods.

Functional renormalization group (FRG)

Functional Renormalization Group (FRG)

What is FRG?: Functional realizations of Wilsonian renormalization group (Wilson & Kogut 1974, Wegner & Houghton 1973,. . .)

Effective couplings flow as a scale parameter k changes:



Wetterich equation

Schwinger functional W_Λ with an IR regulator R_Λ:

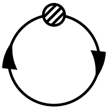
$$\exp(W_\Lambda[J]) = \int \mathcal{D}\phi \exp\left(-S[\phi] - \frac{1}{2}\phi\cdot R_\Lambda\cdot\phi + J\cdot\phi\right).$$

The 1PI effective action $\Gamma_\Lambda$is introduced via the Legendre trans.:

$$\Gamma_\Lambda[\varphi] + \frac{1}{2}\varphi\cdot R_\Lambda\cdot\varphi = J[\varphi]\cdot\varphi - W_\Lambda[J[\varphi]],$$

which obeys the flow equation (Wetterich 1993, Ellwanger 1994, Morris 1994)

$\partial_\Lambda R_\Lambda$

$$\partial_\Lambda\Gamma_\Lambda = \frac{1}{2}$$  .

$[\delta^2\Gamma_\Lambda/\delta\varphi\delta\varphi + R_\Lambda]^{-1}$

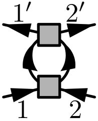
Properties of Γ_Λ: $\Gamma_\Lambda \to S$ as $R_\Lambda \to \infty$, and $\Gamma_\Lambda \to \Gamma$ as $R_\Lambda \to 0$.

Description of BEC without bosonic fields

Scattering physics in vacua

Structures of RG flow of the effective fermion-fermion 4-point coupling $\Gamma_k^{(4)}$:

$$-\partial_k \Gamma_k^{(4)}(P) = \widetilde{\partial}_k$$



$(\widetilde{\partial}_k = \partial_k R_k \frac{\partial}{\partial R_k})$.

In case $a_S > 0$, a dimer pole appears in $\Gamma_k^{(4)}$:

$$\Gamma_k^{(4)}(P) = -\frac{8\pi}{m^2 a_S} \frac{\left(1 + \sqrt{1 + m a_S^2 (iP^0 + \frac{\boldsymbol{P}^2}{4m})}\right)}{2(iP^0 + \boldsymbol{P}^2/4m)}$$

Chemical potential of fermions = half of the binding energy $1/4m_r a_S^2$.

Many-body physics: BEC of dimers

Energy scales: Binding energy $1/ma_S^2 \gg n^{2/3}$
Self-energy correction Σ:

- High-energy $k \gg n^{1/3} \Rightarrow \Sigma \simeq 0$ (RG flow in vacuum)
- Low-energy $k \simeq n^{1/3} \Rightarrow \Sigma$ appears

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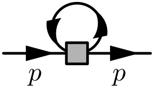
We take into account the self-energy correction in low-energy regions:

$\Sigma(p) =$



$$\simeq n \frac{16\pi/a_S}{ip^0 - \boldsymbol{p}^2 - 1/a_S^2}$$

Superfluid transition temperature : $T_c^{\mathrm{BEC}} \simeq 0.218\varepsilon_F$.

Consequence

Free bosonic picture and its BEC are obtainable only with fermions using FRG.

Summary & Perspectives

Summary:

- Separation of energy scales is a powerful idea in studying physics.
- Without any auxiliary fields, BEC of bound states can be established if we use a nonperturbative way.

Perspectives:

- Taking into account interactions between bound states.
- Application of this formalism to competition of different orders.